Exponential versus linear growth

Focus questions	How fast has the human population grown compared to corn/soybean yield? How might we map these rates into a mathematical model or equation?
Learning target	Students will use graphing software to create a mathematical equation for human population growth and crop yields. Students will compare crop yield and human population growth to ecological models of population dynamics.
Vocabulary	Linear equation, Y intercept, slope, exponential growth, growth value, regression analysis, line of best fit

HS-LS2 Ecosystems: Interactions, Energy, and Dynamics

Performance expectation HS-LS2-2	**Classroom connection:** Students will compare the type of growth in human population and corn and soybean yields over time (linear or exponential) using software to create a mathematical equation. **Classroom connection:** Students will compare population dynamics characteristics to crop and human populations.

Science and engineering practices

Using mathematics and computational thinking	**Classroom connection:** Students will analyze and use data to create a mathematical model. Instead of just observing that the graph looks linear or exponential, students will be able to prove which model is a better representation of growth of the population. **Classroom connection:** Students will compare population data from an ecosystem in nature to the data gathered in this activity.

Disciplinary core ideas

LS2.A: Interdependent Relationships in Ecosystems **LS2.C: Ecosystem Dynamics, Functioning, and Resilience**	**Classroom connection:** Students will be able to use a variety of data sets to revise their mathematical model. I.e. Students may be given three data sets of corn/ soybean production over a 10 year period with results stemming from the introduction of a new technologically enhanced variety, a new fertilizer being used, and a new population data set. **Classroom connection:** Students compare the populations within natural ecosystems to the yields of crops and populations of humans.

Cross-cutting concepts

Scale, Proportion, and Quantity	**Classroom connection:** Students will compare the model and equation of corn and soybean yields vs the model and equation of human population that they have created using graphing software.

Background

Regression is a method used to determine the statistical relationship between a dependent variable and one or more independent variables. They are frequently used by scientists, engineers, and other professionals to predict a result based on a given input. These equations are developed from a set of data obtained through observation or experimentation. There are many types of regression equations, but the simplest one is the linear regression equation. A linear regression equation is simply the equation of a line that is a “best fit” for a particular set of data. This equation may also be used to describe a trend.

A linear regression equation takes the same form as the equation of a line and is often written in the following way: $y = mx + b$, where ‘x’ is the independent variable (the known value) and ‘y’ is the dependent variable (the predicted value). The letters m and b represent constants that describe the y intercept (m) and the slope (b) of the line when x = 0.

The correlation coefficient (r) is a number between 0 and 1 (basically a percentage) that tells you how well the equation actually describes the set of data. The closer the r value is to 1, the more accurate the equation is at representing a linear relationship.

owlcation.com/stem/How-to-Create-a-Simple-Linear-Regression-Equation

Exponential regression is very similar to linear regression, where an attempt is made to arrive at an equation for the line that best fits a set of data, for example, in situations where there is slow growth initially and then a quick acceleration of growth, or in situations where there is rapid decay initially and then a sudden deceleration of decay. The equation used to describe an exponential regression is $y = a \times b^x$.

The line that gives the smallest sum of squared errors is called the regression line. The relative predictive power of an exponential model is denoted by R^2. The value of R^2 varies between 0 and 1. The closer the value is to 1, the more accurate the model is at describing the relationship as exponential.

To learn more about linear and expoential regression, see:

- owlcation.com/stem/How-to-Create-a-Simple-Linear-Regression-Equation
- courses.lumenlearning.com/waymakercollegealgebra/chapter/exponential-and-logarithmic-regression/

In this activity, students will analyze human population growth and crop yield data to determine if a linear relationship or exponential relationship best describes the increase over time.

Prior knowledge

- Students should have an understanding of the difference between a linear function and an exponential function.
- Students should have a general knowledge of how to use the Desmos graphing calculator.

Materials

- A device for access to the internet
- desmos.com
- Student handout
- Data sets (if preferred to give to students)
- Note: Data from seven countries is provided at nourishthefuture.org/go/country-data. The full set of crop and population data is available: crop yields from ourworldindata.org/crop-yields (under 'How have crop yields changed since 1960?'; chart on right side of page; download tab) human population growth from ourworldindata.org/search?q=population+by+country under chart on right download tab.

Teacher preparation

This lesson is designed to take two class periods: Day one to find and complete the data sets in Desmos and Day two to analyze data and discuss possible reasons for the changes they saw in their data due to agricultural advances (i.e. plant genetics (selective breeding advances such hybrids and development of GMOs), farming practices, nutrient requirements, etc.).

This lesson looks at two population types that are human influenced, not controlled by natural predation. This lesson is designed to take two class periods: Day one to find and complete the data sets in Desmos and Day two to analyze data and discuss possible reasons for the changes they saw in their data due to agricultural advances (i.e. plant genetics (selective breeding advances such hybrids and development of GMOs), farming practices, nutrient requirements, etc.). In natural ecosystems predators and prey have a delicate relationship; something that affects the population of one will change the population of the other, especially if the change is sustained for a period of time. The same is true for human/crop relationships. As our population increases, we need to grow more plants for food. If we help to change the plants (the way nature would through evolution in a natural environment) they have a better chance of surviving and thriving, which ultimately feeds us. Examples include GMOs and hybrid technology. Since humans can affect change more quickly than nature, things like advances in mechanical equipment benefit the plants which again, benefit us in the long run, just as predators and prey "one up" each other through time to gain an advantage and make them successful—hence increasing the population.

Students may have looked at ecosystem dynamics that correlate with the stability of populations within an ecosystem. The populations discussed in this lesson do not have the same limits, but carrying capacity is still a moderating force, even though predation is not.

This lesson is designed for students to work in groups of three. The lesson may be turned in as a Google assignment through Schoology or a similar platform.

1. Use Desmos to create an example of the graphs and equations that are required. Create an example to use for demonstration of how to use the system. Teachers should have an understanding of how to create a line for regression for a linear graph and exponential graph. This activity was designed to be used with desmos.com graphing calculator. These youtube clips help you to prepare the mathematical equations using statistics necessary for this activity.
    - Linear regression DESMOS: **youtu.be/zcZaI-xfiFE&t=147s**
    - Exponential regression DESMOS: **youtu.be/3y_GeG88wgw**
2. Examine the data sites in advance in order to help students locate usable data sets. Some data must be downloaded in order to see the historical amounts of crop yields and human populations. Data may be provided to students to be imported into Desmos through a spreadsheet. Additional data sites may be used for country populations.
    a. Population data may need to be reported in 1000s or 100,000s to fit properly on the graph in Desmos. (i.e. 330,000,000 is entered as 330). Be sure students are using consistent units.
    b. Statistics from the following website provide data to analyze on human population and crop yields over time.
    c. **ourworldindata.org/crop-yields**
    d. **ourworldindata.org/world-population-growth**
    e. This activity is focused on the growth of corn and soybean yields as compared to human population growth across the same time frame. Time frames may be shortened to show specific advances in agriculture technology.
3. Create data sets from specific time periods so that students will be able to modify their graphs and equations and compare them to older or newer data sets. Encourage students to investigate what might have caused the data sets to change and how that affects the graphs and equations. An example would be to create or find data about how a change in a plant's genetics (a new hybrid) affected the production in a certain population.

Differentiation

Other ways to connect with students with various needs:

- **Local community:** Students may use data from their town, county, or state. Local farm bureaus or neighboring farmers may share their data on yield.
- **Students with special needs (language/reading):** Students may use data from their town, county, or state. Local farm bureaus or neighboring farmers may share their data on yield.
- **Extra support:** Student groupings should be diverse with learners of varying levels. Students needing extra help or practice may be given extended time to complete the assignment or smaller data sets to analyze.
- **Extensions:** Students work through the following simulation on natural populations and carrying capacity: **sites.nicholas.duke.edu/ecologyapp/modules/population-dynamics/** Students may watch the following videos:
    - Serious Science: History of soy: **youtu.be/pyWMFBbRSoM** which describes the origin of soybeans and their arrival in the United States.
    - Human population through time **youtu.be/PUwmA3Q0_OE** which shows the growth and migration of human populations through time. Students will see how the human population has changed and shifted from area to area over time.

Student handout

Analyze the data

1. Looking at the r value compared to R^2 in the corn growth, is your country's corn growth growing exponentially, linear, or neither? Please explain your answer using the r or R^2 value.

Answers may vary. The closer the r value is to 1 means the graph is more accurate. So if the exponential r^2 = 0.98 and the linear equation r = 0.73, this means that the graph is more of an exponential growth than a linear growth. If both values are not close to 1 then it can be listed as neither.

2. Looking at the r value compared to R^2 in the corn growth, is your country's corn growth growing exponentially, linear, or neither? Please explain your answer using the r or R^2 value.

Answers may vary. The closer the r value is to 1 means the graph is more accurate. So if the exponential r^2 = 0.98 and the linear equation r = 0.73, this means that the graph is more of an exponential growth than a linear growth. If both values are not close to 1 then it can be listed as neither.

Day 2

1. After reviewing your graphs and equations from the extra data sets you were given to analyze, what conclusions can you make about:
   a. ...the new hybrid that was used?

Answers will vary. Students may find that the production graph was higher over time and more exponential depending on the length of time the data set covers. Technological advances may lead to exponential growth.

   b. ...the new population numbers that you were given?

Answers will vary. Students may find that the population graph was higher over time and more exponential depending on the length of time the data set covers. Pandemics and famine or other natural disasters may lead to differing rates of growth.

   c. ...the new corn production numbers after the new fertilizer was used?

Answers will vary. Students may find that the production graph was higher over time and more exponential depending on the length of time the data set covers. Technological advances may lead to exponential growth.

2. List at least 3 other agricultural possibilities that would change the graphs and equations over time—such as #1 and #3 in question 3 did.

Possibilities include new farming techniques, insecticides, organic fertilizers, biotechnology practices, technologically advanced machinery, etc.

3. How would this change the carrying capacity of the land for the crops?

If the graphing analysis showed that production was increased then carrying capacity for crops was increased.

4. How would this change the carrying capacity of the country for humans?

If the graphing analysis showed that population was increasing while the production also increased, then the carrying capacity for humans was also increased.

Assessments

Rubric for assessment

Skill	Beginning	Satisfactory	Exemplar
Student was able to input a data set and visualize the data in graph form.	Student did not have the complete data set on Desmos. They had some of the points but did not finish all the data points needed to complete the task. Some points were placed properly but others were incorrectly placed or labelled.	Student had all data points in the Desmos table. No points were skipped and they were all labelled correctly in the table.	Student went above and beyond and put in more data than necessary.
Student was able to analyze the equation and data using the r values.	Student created the two equations correctly, but were unable to read the r value correctly to have the right graph to identify the relationship as linear or exponential. No explanation provided.	Student created two equations correctly and selected the appropriate equation for the given data set and graph. The explanation was vague and did not provide proof of understanding of the r value of the data.	Student was able to create two equations correctly and selected the appropriate equation for the given data set and graph. The explanation proved that they had an understanding of the r value and how to apply to the data set.
Student will be able to compare the graphs and analyze what the relationship is between human population and crop yield as the variables change.	Very little analysis of the different variables used for yield and human population are used. Most variables are not addressed. No references to the graphs used in the analysis are included.	Some analysis of the different variables used for yield and human population is included. Not all variables are addressed. No references to the graphs used in the analysis are included.	Analysis of the different variables used for production and population is included. All variables are addressed and analyzed. References to the graphs used in the analysis are included.

Rubric for self-assessment

Skill	Yes	No	Unsure
I was able to put the data sets into Desmos.			
I was able to create two mathematical models using my data set and Desmos.			
I was able to compare the graphs and analyze what the relationship is between human population and crop yield as the variables changed.			