Exponential versus linear growth

Focus questions	How fast has the human population grown compared to corn/soybean yield? How might we map these rates into a mathematical model or equation?
Vocabulary	Linear equation, Y intercept, slope, exponential growth, growth value, regression analysis, line of best fit

Regression equations are frequently used by scientists, engineers, and other professionals to predict a result based on a given input. These equations are developed from a set of data obtained through observation or experimentation. There are many types of regression equations, but the simplest one is the linear regression equation. A linear regression equation is simply the equation of a line that is a "best fit" for a particular set of data. This equation may also be used to describe a trend.

A linear regression equation takes the same form as the equation of a line and is often written in the following way: $y = mx + b$, where 'x' is the independent variable (the known value) and 'y' is the dependent variable (the predicted value). The letters m and b represent constants that describe the y intercept (m) and the slope (b) of the line when x = 0.

The correlation coefficient (r) is a number between 0 and 1 (basically a percentage) that tells you how well the equation actually describes the set of data. The closer the r value is to 1, the more accurate the equation is at representing a linear relationship.

Exponential regression is very similar to linear regression, where an attempt is made to arrive at an equation for the line that best fits a set of data, for example, in situations where there is slow growth initially and then a quick acceleration of growth, or in situations where there is rapid decay initially and then a sudden deceleration of decay. The equation used to describe an exponential regression is $y = a \times b^x$.

The line that gives the smallest sum of squared errors is called the regression line. The relative predictive power of an exponential model is denoted by R^2. The value of R^2 varies between 0 and 1. The closer the value is to 1, the more accurate the model is at describing the relationship as exponential.

In this activity, students will analyze human population growth and crop yield data to determine if a linear relationship or exponential relationship best describes the increase over time.

Procedure

Day 1

Complete this activity in groups of 3: one student creates the corn graph and equations, one student creates the soybean graph and equations, and the other student creates the population graph and equations. Open Desmos (`desmos.com/calculator`). Use the data sets assigned to you to create your regression equations.

1. Which country are you choosing to determine crop yield growth and human population growth?

2. Create your crop yield graph in Desmos. Please have your x value start at 0. Record data for every 3 years of crop data (so x = 0 for your first data point, x = 3 for your next data point, etc…). You should have 19 or 20 data points on your graph. If you do not based on your chosen data set, please record closer intervals (every 2 years or every year).

3. Create your human population graph. Please have your x value start at 0. Record data for every 10 years of human population data. You should have 19 or 20 data points on your graph.

4. Please export the images of your graphs to be turned in.

5. After putting your data sets into Desmos, now is the time to create an exponential and linear equation for each data set. Please fill in the values.
   a. Crop Growth Exponential - Corn
      - Equation:
      - r^2 =
      - a=
      - b=
   b. Crop Growth Linear - Corn
      - Equation:
      - r=
      - m=
      - b=

c. Crop Growth Exponential - Soybeans
   - Equation:
   - r^2 =
   - a=
   - b=
d. Crop Growth Linear - Soybeans
   - Equation:
   - r=
   - m=
   - b=
e. Population Growth Exponential
   - Equation:
   - r^2 =
   - a=
   - b=
f. Population Growth Linear
   - Equation:
   - r=
   - m=
   - b=

Analyze the data

1. Looking at the r value compared to R^2 in the corn growth, is your country's corn growth growing exponentially, linear, or neither? Please explain your answer using the r or R^2 value.

2. Looking at the r value compared to R^2 in the corn growth, is your country's corn growth growing exponentially, linear, or neither? Please explain your answer using the r or R^2 value.

Day 2

1. After reviewing your graphs and equations from the extra data sets you were given to analyze, what conclusions can you make about:
   a. ...the new hybrid that was used?

   b. ...the new population numbers that you were given?

   c. ...the new corn production numbers after the new fertilizer was used?

2. List at least 3 other agricultural possibilities that would change the graphs and equations over time—such as #1 and #3 in question 3 did.

3. How would this change the carrying capacity of the land for the crops?

4. How would this change the carrying capacity of the country for humans?

Rubric for self-assessment

Skill	**Yes**	**No**	**Unsure**
I was able to put the data sets into Desmos.			
I was able to create two mathematical models using my data set and Desmos.			
I was able to compare the graphs and analyze what the relationship is between human population and crop yield as the variables changed.			